Hmwk 15

1. $$my'' + ke^{-\alpha t} y = 0$$

$$s = \frac{2}{\alpha}\sqrt{\frac{k}{m}}\, e^{-\alpha t/2}$$

1a) $$e^{-\alpha t/2} = \frac{\alpha}{2}\sqrt{\frac{m}{k}}\, s$$

$$\Rightarrow e^{-\alpha t/2} = \frac{\alpha^2}{4}\frac{m}{k} s^2$$

$$\frac{d}{dt} = \frac{d}{ds}\frac{ds}{dt} = \underbrace{-\sqrt{\frac{k}{m}}\, e^{-\alpha t/2}}_{\frac{ds}{dt}} \frac{d}{ds}$$

(but we know that $e^{-\alpha t/2} = \frac{\alpha^2}{4}\frac{m}{k} s^2$)

$$\Rightarrow \boxed{\frac{d}{dt} = -\frac{\alpha}{2} s \frac{d}{ds}}$$

$$\frac{d^2}{dt^2} = \frac{d}{dt}\left(\frac{d}{dt}\right) = \left(-\frac{\alpha}{2} s \frac{d}{ds}\right)\left(-\frac{\alpha}{2} s \frac{d}{ds}\right)$$

$$= \frac{\alpha^2}{4} s \frac{d}{ds}\left(s \frac{d}{ds}\right)$$

(chain rule)

$$= \frac{\alpha^2}{4} s \left(1 \cdot \frac{d}{ds} + s \frac{d^2}{ds^2}\right)$$

$$\boxed{\frac{d^2}{dt^2} = \frac{\alpha^2}{4}\left(s \frac{d}{ds} + s^2 \frac{d^2}{ds^2}\right)}$$

1b Plugging in:

$$\cancel{M}\,\cancel{\frac{\alpha^2}{4}}\left(s^2 y''(s) + s\,y'(s)\right) + \cancel{k}\left(\cancel{\frac{\alpha^2}{4}}\,\cancel{\frac{m}{k}}\,s^2\right) y = 0$$

(dividing through)

$$\boxed{\Rightarrow s^2 y''(s) + s\,y'(s) + \left(s^2 - 0^2\right) y = 0}$$

$$\boxed{1c \Rightarrow \nu = 0}$$

1d $\Rightarrow y(s) = c_1 J_0(s) + c_2 Y_0(s)$

$$\boxed{\Rightarrow y(t) = c_1 J_0\left(\frac{2}{\alpha}\sqrt{\frac{k}{m}}\,e^{-\frac{\alpha t}{2}}\right) + c_2 Y_0\left(\frac{2}{\alpha}\sqrt{\frac{k}{m}}\,e^{-\frac{\alpha t}{2}}\right)}$$

2 a) Using the dfn. on page 189

$\theta(1)$: 1

$\theta(t^1)$: 0

$\theta(t^2)$: $\frac{-t^2}{4}$ where $t = 2w\sqrt{\frac{x}{g}}$

$\theta(t^3)$: 0

$\theta(t^4)$: $\frac{t^4}{64}$

$$\Rightarrow \boxed{J_0\left(2w\sqrt{\frac{x}{g}}\right) = 1 - \frac{w^2 x}{g} + \frac{w^4}{4g^2}x^2}$$

Using Quadratic formula

where $a = \frac{w^4}{4g^2}$, $b = -\frac{w^2}{g}$

and $c = 1$

$$\Rightarrow \boxed{x_{1,2} = \frac{2g}{w^2}}$$

2c) since w is the rate,

and $x_{1,2}$ is $\Theta\left(\frac{1}{w^2}\right)$, if

I double w then the

root will change by a factor

of $\boxed{\frac{1}{4}}$ $\left(\frac{1}{w^2} = \frac{1}{(2)^2} = \frac{1}{4}\right)$