Hmwk 13

1a) $\sum_{m=0}^{\infty} (m+1)m\, x^m$

$$R = \frac{1}{\lim_{m\to\infty} \left|\frac{a_{m+1}}{a_m}\right|} = \frac{1}{\lim_{m\to\infty} \left|\frac{(m+2)(m+1)}{(m+1)(m)}\right|} = \frac{1}{1} = 1$$

$$\Rightarrow \boxed{R = 1}$$

1 b) $R = \frac{1}{\lim_{m\to\infty} \left|\frac{a_{m+1}}{a_m}\right|}$

$= \frac{1}{\lim \left|\frac{a_{m+2}}{\ldots}\right|}$ (since y_1 and y_2 only have every other term for solution)

$\lim_{m\to\infty} \left| \quad a_m \quad \right|$ to legendre eqn)

$$= \lim_{m\to\infty} \left| \frac{1}{\dfrac{-\dfrac{(n-m-2)(n+m+3)}{(m+4)(m+3)}}{-\dfrac{(n-m)(n+m+1)}{(m+2)(m+1)}}} \right|$$

$$= \lim_{m\to\infty} \left| \frac{1}{\dfrac{-(m+2)(m+1)(n-m-2)(n+m+3)}{(m+4)(m+3)(n-m)(n+m+1)}} \right|$$

$$= \lim_{m\to\infty} \left(\frac{1}{\dfrac{m^4}{m^4}} \right)$$

$= \quad 1$

$R = 1$

1c) $1 - x^2 = 0 \Rightarrow x = \pm 1$

are locations of singular points!

2a) $y'' - xy = 0$

$$y = \sum_{m=0}^{\infty} a_m x^m$$

$$\Rightarrow \quad y'' = \sum_{m=2}^{\infty} a_m (m)(m-1) x^{m-2}$$

$$\Rightarrow \sum_{m=2}^{\infty} a_m (m)(m-1) x^{m-2} - \sum_{m=0}^{\infty} a_m x^{m+1} = 0$$

$\mathcal{O}(x^0)$: $a_2 (2)(1) = 0$

$\mathcal{O}(x^1)$: $a_3 (3)(2) - a_0 = 0$

$O(x^2)$: $a_4 (4)(3) - a_1 = 0$

$\vdots$

$O(x^m)$: $a_m (m)(m-1) - a_{m-3} = 0$

$$a_m = \frac{a_{m-3}}{m(m-1)} \text{ for } m > 3$$

& $a_0 = a_0$

$a_1 = a_1$

$a_2 = 0$

2b) $a_3 = \dfrac{a_0}{3 \cdot 2} = \dfrac{a_0}{6}$

$$a_4 = \frac{a_1}{4\cdot 3} = \frac{a_1}{12}$$

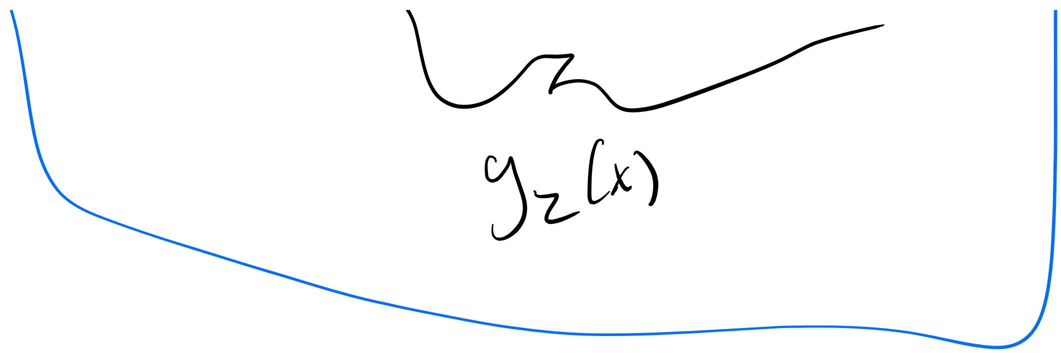
$$a_5 = \frac{a_2}{5\cdot 4} = 0$$

$$a_6 = \frac{a_3}{6\cdot 5} = \frac{a_0}{180}$$

$$a_7 = \frac{a_4}{7\cdot 6} = \frac{a_1}{504}$$

$$\Rightarrow y(x) = a_0 \overbrace{\left(1 + \frac{x^3}{6} + \frac{x^6}{180}\right)}^{y_1(x)}$$

$$+ a_1 \left(x + \frac{x^4}{12} + \frac{x^7}{504}\right)$$


$y_2(x)$