

Hmwork 11

$$(a) \quad h'' = 64 \delta(t-1) - gH(t-1)$$

$$h(0) = 0, \quad h'(0) = 0$$

$$\mathcal{L}(h'') = s^2 H(s) - \underbrace{s h(0)}_0 - \underbrace{h'(0)}_0$$

$$64 \mathcal{L}(\delta(t-1)) = 64 e^{-s}$$

$$-g \mathcal{L}(H(t-1)) = -g \frac{e^{-s}}{s}$$

$$\Rightarrow H(s) = \frac{e^{-s} \left(64 - \frac{g}{s} \right)}{s^2}$$

$$= e^{-s} \left(\frac{64}{s^2} - \frac{g}{s^3} \right)$$

$$1b) \quad h(t) = 64 \mathcal{L}^{-1}\left(e^{-s} \frac{1}{s^2}\right) - g \mathcal{L}^{-1}\left(e^{-s} \frac{1}{s^3}\right)$$

(s-shifting)

$$= 64 (t-1) H(t-1) - g \frac{1}{2} (t-1)^2 H(t-1)$$

$$= H(t-1) (t-1) \left(64 - \frac{g}{2} (t-1)\right)$$

$$1c \quad \frac{d}{dt} \left((t-1) \left(64 - \frac{g}{2} (t-1)\right) \right) = 0$$

$$\Rightarrow t = 3$$

$$\Rightarrow h(3) = 64$$

$$2a \quad y(t) + 4 \int_0^t y(\tau) (t-\tau) d\tau = 2t$$

$$\Rightarrow y + 4(y * t) = 2t$$

$$\Rightarrow \mathcal{L}(y) + 4\mathcal{L}(y * t) = 2\mathcal{L}(t)$$

$$\Rightarrow Y(s) + 4\mathcal{L}(y)\mathcal{L}(t) = \frac{2}{s^2}$$

$$\Rightarrow Y + 4Y \frac{1}{s^2} = \frac{2}{s^2}$$

$$\Rightarrow Y = \frac{2}{s^2 \left(1 + \frac{4}{s^2}\right)}$$

$$= \frac{2}{s^2 + 4}$$

$$y(t) = \int_0^t y(\tau) \sin(2(t-\tau)) d\tau + \sin(2t)$$

$$\mathcal{L}(y(t)) - \mathcal{L}(y(t) \sin(2t)) = \mathcal{L}(\sin(2t))$$

$$Y - Y \cdot \frac{2}{s^2+4} = \frac{2}{s^2+4}$$

$$Y = \frac{2}{(s^2+4) \left(1 - \frac{2}{s^2+4}\right)} = \frac{2}{s^2+4-2}$$

$$= \frac{2}{s^2+2} = \sqrt{2} \frac{\sqrt{2}}{s^2+2}$$

$$\Rightarrow y(t) = \sqrt{2} \mathcal{L}^{-1} \left(\frac{\sqrt{2}}{s^2+2} \right)$$

$$= \sqrt{2} \sin(\sqrt{2}t)$$
