1(a) $$LI'' + RI' + \frac{1}{C} I = E'(t)$$

By Integrating the given eqn, we get

$$LI' + RI + \frac{1}{C}\int I = E(t)$$

$$\Rightarrow LI' + RI + \frac{1}{C} Q = 100 \sin(\omega t)$$

we have $I(0) = Q(0) = 0$,

then $LI'(0) = 0$

But, $L = 1\,H \neq 0$

Thus, $\underline{\underline{I'(0) = 0}}$

1 (b) $LI'' + RI' + \frac{1}{C} I = 0$

$L = 1H$, $R = 18\Omega$, $C = 12.5 \times 10^{-3} F$

$I'(0) = I(0) = 0$

The characteristic :

$L\lambda^2 + R\lambda + \frac{1}{C} = 0$

$\Rightarrow \lambda^2 + 18\lambda + \frac{1}{12.5 \times 10^{-3}} = 0$

$\Rightarrow \lambda^2 + 18\lambda + 80 = 0$

$\Rightarrow (\lambda + 8)(\lambda + 10) = 0$

$\Rightarrow \lambda = -8$ or -10

$\Rightarrow I_h(t) = c_1 e^{-8t} + c_2^{-10t}$

1(c) $LI'' + RI' + \frac{1}{C} I = E'(t)$

$\Rightarrow I'' + 18I' + 80I = 1000\cos(10t)$

Assume $I(t) = A\cos(\omega t) + B\sin(\omega t)$

$I'(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$

$I''(t) = -A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t)$

$\Rightarrow \left(-A\omega^2\cos(\omega t) - B\omega^2\sin(\omega t)\right)$

$+ 18\left(-A\omega\sin(\omega t) + B\omega\cos(\omega t)\right)$

$+ 80\left(A\cos(\omega t) + B\sin(\omega t)\right)$

$= 1000\cos(10t)$

$\Rightarrow \omega = 10$

$$\Rightarrow \begin{cases} -100A + 180B + 80A = 1000 \\ -100B - 180A + 80B = 0 \end{cases}$$

$$\Rightarrow \begin{cases} -A + 9B = 50 \\ -B - 9A = 0 \end{cases}$$

$$\Rightarrow A = -\frac{25}{41} \quad \& \quad B = \frac{225}{41}$$

Thus, $I_p(t) = -\frac{25}{41}\cos(10t) + \frac{225}{41}\sin(10t)$

$$I(t) = I_h(t) + I_p(t)$$

$$= c_1 e^{-8t} + c_2 e^{-10t}$$

$$- \frac{25}{41}\cos(10t) + \frac{225}{41}\sin(10t)$$

By the Initial conditions, we have

$$I(0) = c_1 + c_2 - \frac{25}{41} = 0$$

$$I'(0) = -8c_1 - 10c_2 + \frac{2250}{41} = 0$$

$$\Rightarrow c_1 = -\frac{1000}{41}, \quad c_2 = 25$$

$$\Rightarrow I(t) = -\frac{1000}{41} e^{-8t} + 25 e^{-10t}$$

$$- \frac{25}{41}\cos(10t) + \frac{225}{41}\sin(10t)$$

1(d) $I(t) = c_1 e^{-8t} + c_2 e^{-10t}$

$$- \frac{25}{41}\cos(10t) + \frac{225}{41}\sin(10t)$$

$\Rightarrow I(t) \to -\frac{25}{41}\cos(10t) + \frac{225}{41}\sin(10t)$

as $t \to \infty$

Note. $\lim_{t\to\infty} e^{-kt} = 0$ (if $k>0$)

1(e) This is equivalent to a spring-mass system with damping

$$LI' + RI + \frac{1}{C}\int I(t) = E(t)$$

$$\Leftrightarrow m y'' + c y' + k y = F(t)$$

$L \longleftrightarrow$ mass m

$R \longleftrightarrow$ damping c

$1/C \longleftrightarrow$ spring constant k

$E \longleftrightarrow$ external force F

$I \longleftrightarrow$ velocity y'