1(a) $\begin{cases} m x'' = -\rho_w g A_c x \\ x'(0) = 0 \end{cases}$

$\Rightarrow \begin{cases} m x'' + \rho_w g A_c x = 0 \\ x'(0) = 0 \end{cases}$

The characteristic equation:

$$m \lambda^2 + \rho_w g A_c = 0$$

$$\Rightarrow \lambda = \pm \sqrt{\frac{-\rho_w g A_c}{m}} = \pm \sqrt{\frac{\rho_w g A_c}{m}}\, i$$

$$\Rightarrow x(t) = c_1 \cos\left(\sqrt{\frac{\rho_w g A_c}{m}}\, t\right) + c_2 \sin\left(\sqrt{\frac{\rho_w g A_c}{m}}\, t\right)$$

$x'(0) = 0 \Rightarrow c_2 = 0$

$$\therefore\ x(t) = c_1 \cos\left(\sqrt{\frac{\rho_w g A_c}{m}}\, t\right)$$

1(b) $\rho_w = 1\ g/cm^3$

$r = 8\ cm$

$g = 981\ cm/s^2$

$$\Rightarrow x(t) = c_1 \cos\left(\sqrt{\frac{1 \cdot 981 \cdot \pi 8^2}{m}}\, t\right)$$

we know that it oscillates with a period of 0.5 sec.

$$\text{period} = \frac{2\pi}{\sqrt{\frac{1 \cdot 981 \cdot \pi 8^2}{m}}} = 0.5$$

$$\Rightarrow 4\pi = \sqrt{\frac{1 \cdot 981 \cdot \pi 8^2}{m}}$$

$$\Rightarrow 16\pi^2 = \frac{981 \cdot 64\pi}{m}$$

$$\Rightarrow m = \frac{981 \cdot 4}{\pi}$$

1(c) we verified that period $\propto \sqrt{m}$

$$\text{frequency} = \frac{1}{\text{period}} \propto \frac{1}{\sqrt{m}}$$

i) if $m \uparrow$, frequency (oscillation) $\downarrow$

ii) if $m \downarrow$, frequency (oscillation) $\uparrow$

1(d) The period is the same.

it does not depend on the amplitude.