1(a) $s'' = \sqrt{1+(s')^2}$

$\Rightarrow$ Let $s' = x$.

Then the given eqn can be written as

$$x' = \sqrt{1+x^2}.$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{1+x^2}$$

$$\frac{1}{\sqrt{1+x^2}}\,dx = dt$$

$$\Rightarrow \sinh^{-1}(x) = t + c_1$$

$$\Rightarrow x = \sinh(t+c_1) = s'$$

$$\Rightarrow \frac{ds}{dt} = \sinh(t+c_1)$$

$$\Rightarrow s = \cosh(t+c_1) + c_2$$

$$\therefore \quad s(t) = \cosh(t+c_1) + c_2$$

(cb)

$(-1,0) \Rightarrow \quad 0 = \cosh(-1+c_1) + c_2$

$(1,0) \Rightarrow \quad 0 = \cosh(1+c_1) + c_2$

$\Rightarrow \quad \cosh(-1+c_1) = \cosh(1+c_1)$

By the symmetry, $c_1 = 0$

$\Rightarrow \quad 0 = \cosh(-1) + c_2 \Rightarrow c_2 = -\cosh(-1)$

$\therefore \quad s(t) = \cosh(t) - \cosh(-1)$

or $= \cosh(t) - \cosh(1)$

$\Rightarrow s'(t) = 0$

$\Rightarrow \sinh'(t) = 0 \Rightarrow t = 0$

$s''(0) = \cosh(0) > 0$

$\therefore t = 0$ is the minimum

$$\boxed{s(0) = \cosh(0) - \cosh(-1)}$$

1(c)

$(-1,0) \Rightarrow 0 = \cosh(-1+c_1) + c_2$

$(1,1) \Rightarrow 1 = \cosh(1+c_1) + c_2$

$\Rightarrow \cosh(-1+c_1) = \cosh(1+c_1) - 1$

$$\Rightarrow \frac{e^{-1+c_1} + e^{1-c_1}}{2} = \frac{e^{1+c_1} + e^{-1-c_1}}{2} - 1$$

$\Rightarrow$ Multiply by e^{c_1}

$$\frac{e^{-1+2c_1} + e}{2} = \frac{e^{1+2c_1} + e^{-1}}{2} - e^{c_1}$$

$$\Rightarrow \frac{1}{2}\left(e - \frac{1}{e}\right) e^{2c_1} - e^{c_1} + \frac{1}{2}\left(\frac{1}{e} - e\right) = 0$$

$$\Rightarrow e^{c_1} = \frac{1 + \sqrt{1 - \left(e - \frac{1}{e}\right)\left(\frac{1}{e} - e\right)}}{e - \frac{1}{e}}$$

$$= \frac{1 + \sqrt{1 - \left(1 - e^2 - \frac{1}{e^2} + 1\right)}}{e - \frac{1}{e}}$$

$$= \frac{1 + \sqrt{e^2 + \frac{1}{e^2} - 1}}{e - \frac{1}{e}}$$

$$\Rightarrow c_1 = \ln\left(\frac{1+\sqrt{e^2-\frac{1}{e^2}-1}}{e-\frac{1}{e}}\right)$$

$$\text{or} = \ln\left(\frac{e+\sqrt{e^4-e^2-1}}{e^2-1}\right)$$

$$\Rightarrow c_2 = -\cosh(-1+c_1)$$

$$\therefore \quad S(t) = \cosh(t+c_1) - \cosh(-1+c_1)$$

$$\Rightarrow S'(t) = \sinh(t+c_1) = 0 \Rightarrow t = -c_1$$

$$S''(t) = \cosh(t+c_1) > 0$$

$\Rightarrow$ $t = -c_1$ is the minimum