

(General Method)

$$|e_n| = k |e_{n-1}|^r$$

Suppose you have errors at each iteration (n): $\vec{e} = \langle e_0, e_1, \dots, e_n \rangle$
 r is the order (1 is linear, 2 is quadratic, ...)

$$\Rightarrow |e_n| = k |k |e_{n-1}|^r|^r = k^{r+1} |e_{n-1}|^{r^2}$$

(Repeat)

$$= \dots = k^{r(n-1)+1} |e_0|^{r^n}$$

$$\star \left\{ \Rightarrow \log(|e_n|) = (r(n-1)+1) \log k + r^n \underbrace{\log(|e_0|)}_{c_2}$$

$$\Rightarrow \log(|e_{n+1}|) - \log(|e_n|) = r \log k + c_2(r-1)r^n$$

$$\Rightarrow \underbrace{\log(\log(|e_{n+1}|) - \log(|e_n|))}_{f_1(n)} = n \log(r) + c_3$$

$$c_3 = \log(r \log k) + \log(c_2(r-1))$$

Thus the slope (m) of $f_1(n)$ versus n is $\log(r)$.

~~Therefore,~~ e^m is the order

(special case: Bisection)

Assume you did the prior work

(or I tell you in advance) $r=1$.
that bisection is linear

$$\star \Rightarrow \underbrace{\log(|e_n|)}_{f_2(n)} = n \log(k) + \log(|e_0|)$$

Thus, the slope (m) of $f_2(n)$ versus n is $\log(k)$

Therefore, e^m is ~~the~~ k .

