Assignment 9

Math 528L Fall 2021

Due 10/31 (Sunday/Spooky Time at The Stroke of Midnight

1 Spring Kings

(a) Solve and plot the damped harmonic equation with forced vibrations:

$$mx'' + cx' + kx = F(t)$$

for $F(t) = F_0 \cos \omega_0 t$ with $\omega_0 = \sqrt{k/m}$ using the 2nd order Runge Kutta method. For comparison, use $dt = .01$, final time $T = 15$, $k = m = c = F_0 = 1$, and for initial values, use $x(0) = 1$ and $x'(0) = 0$ (but make sure to let the user define these parameters and have fun with it.) Also solve with $\omega_0 = 2$. Why do these look different?

(b) Solve and plot the two problems above analytically in mathematica using DSolve. Again, let the user define these parameters.

2 Systemic Epidemic

(c) Solve the SIR problem introduced earlier:

$$\begin{pmatrix} S' \\ I' \\ R' \end{pmatrix} = \begin{pmatrix} -\alpha \frac{SI}{S+I+R} \\ \alpha \frac{SI}{S+I+R} - \beta I \\ \beta I \end{pmatrix}, \quad \begin{pmatrix} S(0) \\ I(0) \\ R(0) \end{pmatrix} = \begin{pmatrix} S_0 \\ I_0 \\ R_0 \end{pmatrix}$$

using your favorite method we've talked about (I would recommend Euler for simplicity) and your own parameters. What happens to the dynamics of the populations?

Some references to look at :

https://community.wolfram.com/groups/-/m/t/1920119

https://en.wikipedia.org/wiki/Compartmental_models_in_epidemiology