Assignment 11

Math 528L Fall 2021

Due November 23rd

1 Heat Equation

We are finally making that graphic on the course website! Time to learn how to solve the classic PDE, the heat equation!

(a) Provide a numerical solution $u(x,t)$ to the heat equation $u_t = ku_{xx}$ on the domain $x \in [0, L]$ and $t \in [0, T]$. The initial condition is of the form: $u(x,0) = A\sin(\alpha x) + B\sin(\beta x)$ such that $\alpha = n\pi/L$ and $\beta = m\pi/L$ with boundary conditions: $u(0,t) = u(0,0)$ and $u(L,t) = 0$. Also note that the stability condition is that $k\dfrac{\Delta t}{\Delta x^2} < \dfrac{1}{2}$. Note, you can use a for loop or use the following matrix for evolving in time:

A = gallery('tridiag',a1,a2,a3);

(b) The exact solution to this problem is, $u(x,t) = Ae^{-tk\alpha^2}\sin(\alpha x) + Be^{-tk\beta^2}\sin(\beta x)$. Plot the exact and numerical solution to the problem above at the final time. Also plot the error at the final time. What do you notice about the shape of the error?

(c) Suppose you have something with high frequency and low frequency. By testing with your code, which decays quicker? Justify your answer via the analytic solution.